(4.25) Find abs. max & min values of $g(s) = 2^{s+3} - \dfrac{2^{3s-4}}{3}$ on $[0, 4]$.

Cr. pts: $g'(s) = (\ln 2) 2^{s+3} - \frac{1}{3} (\ln 2) 2^{3s-4} \cdot 3$

$= \ln(2) \cdot 2^{s+3} - (\ln 2) \cdot 2^{3s-4} = 0$

$\Rightarrow \ln(2) \left(2^{s+3} - 2^{3s-4} \right) = 0$

$\frac{}{\ln(2)} \qquad \frac{}{\ln(2)}$

$\Rightarrow 2^{s+3} = 2^{3s-4}$

$\Rightarrow (s+3) = (3s-4)$

$\Rightarrow 7 = 2s \Rightarrow \boxed{s = \frac{7}{2}}$.

s	$g(s) = 2^{s+3} - 2^{3s-4}/3$
0	$\rightarrow$
7/2	$\rightarrow$ 60.3398 $\leftarrow$ max value
4	$\rightarrow$ 7.9792 $\leftarrow$ min value.

Example. A fence is built in a rectangular shape on the side of a house, so that one side of the rectangle is along the house. If there is 200 ft of fencing material, what design produces the largest area? What is that area?

$200 = 2W + L$ (feet) info given

Want maximum area $\Rightarrow$ Function $= A$ (area).

$$A = LW$$

need this in 1-variable so we can find the max.

$L = 200 - 2W$

$\Rightarrow A = (200 - 2W)W$ ← function of W.

$$\boxed{A(w) = 200w - 2w^2}$$ our function.

<u>Interval</u>: What is the smallest/largest possible values of w?

$$\boxed{0 \le w \le 100} \text{ (ft.)}$$

Now, let's do the math:

Find the maximum of $A(w)$ on $[0, 100]$.

<u>Math</u>: $A'(w) = 200 - 4w = 0$

$200 = 4w \Rightarrow w = 50$ ft.

only cr pt.

closed interval

w	$A(w) = 200w - 2w^2$
0	0
50	$10000 - 5000 = 5000$
100	$20000 - 20000 = 0$.

abs max value

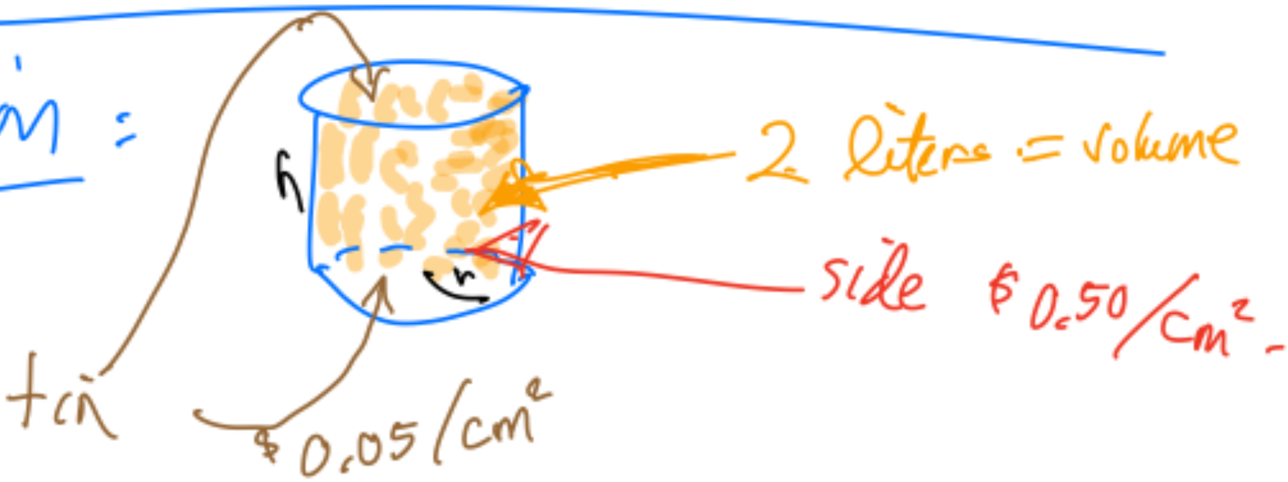
$w = 50 \Rightarrow A(w) = 5000$ ft^2 is max.

$L = 200 - 2w = \underline{100\text{ft}}$.

Best design $\boxed{w = 50\text{ft}, L = 100\text{ft}}$.

$\boxed{\text{Example}}$ Aman has a business that manufactures vienna sausages and puts them in cans. He wants to design a can that holds 2 liters of vienna sausage. The can he will use has metal (tin) on the top & bottom, that costs \$0.05 per square centimeter, and the cylindrical part (sides) is made out of heavy-duty cardboard, which costs \$0.50 per square centimeter. How should Aman design the can so that it holds all the vienna sausages but costs the least amount of money? How much will each can cost?

Solution :



What is the function? Cost ← want to minimize it.

C

$$C = \underbrace{\overset{\$/cm^2}{(0.05)}\cdot 2\pi r^2}_{\text{cost of top + bottom}} + \overset{\$/cm^2}{(\$0.50)}\cdot \underbrace{2\pi r h}_{\text{area of side}}$$

Extra Info:

2 liters = Volume = $\pi r^2 h$

1 liter = 1000 ml = 1000 cm³

cm: $\boxed{2000 = \pi r^2 h} \Rightarrow h = \dfrac{2000}{\pi r^2}$

$$C(r) = (0.05)\cdot 2\pi r^2 + (.50)2\cancel{\pi} r\left(\frac{2000}{\cancel{\pi} r^2}\right)$$

$$\Rightarrow \boxed{C(r) = 0.1\pi r^2 + \frac{2000}{r}}$$ Want minimum

interval: $\boxed{0 < r < \infty}$

Need to look for cpts: $C'(r) = 0$

Need $\lim_{r\to\infty} C(r) = \lim_{r\to\infty} 0.1\pi r^2 + \frac{2000}{r} = \boxed{\infty}$

$0.1\pi r^2 \to \infty$, $\frac{2000}{r} \to 0$

$$\lim_{r\to 0^+} C(r) =$$

$$= \lim_{r\to 0^+} 0.1\pi r^2 + \frac{2000}{r} = \boxed{\infty}$$

$0.1\pi r^2 \to 0$, $\frac{2000}{r} \to \infty$

$$C'(r) = 2(0.1\pi)r + (2000)\frac{-1}{r^2} = 0$$

$$2(0.1\pi)r = \frac{2000}{r^2}$$

$$r^3 = \frac{2000}{2(0.1\pi)}$$

$$\Rightarrow r = \sqrt[3]{\frac{2000}{0.2\pi}}$$

$$= \cancel{6.86\text{ cm}}\ 14.7\text{cm}$$ r

$$h = \frac{2000}{\pi r^2} = \frac{2000}{\pi(\cancel{6.86})^2} = \cancel{13.5}\ 2.95\text{cm}$$ h

14.7

best design!

$$C = \$203.94$$